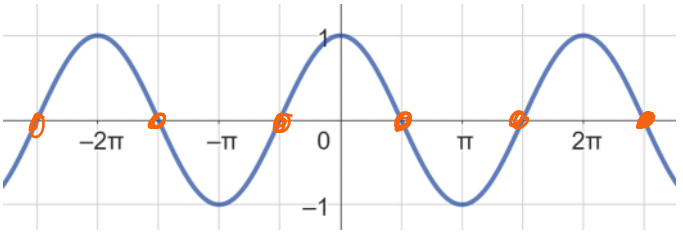
Warm-up!

Periodic Graphs Day 3
- Sinusoidal Graphs

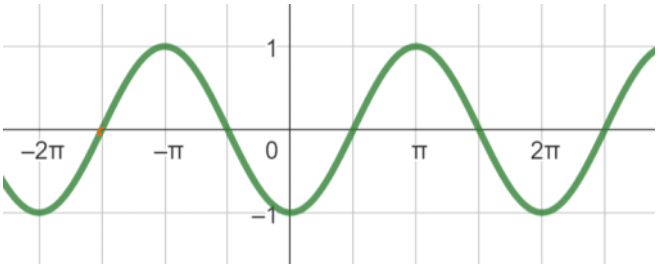
Homework: Sinusoidal Graphs HW

Quiz Class after Next

To the right is the graph of $y = \cos x$



Apply your understanding of transformation of functions to find an equation for the graph to the right.



$y = -\cos x$

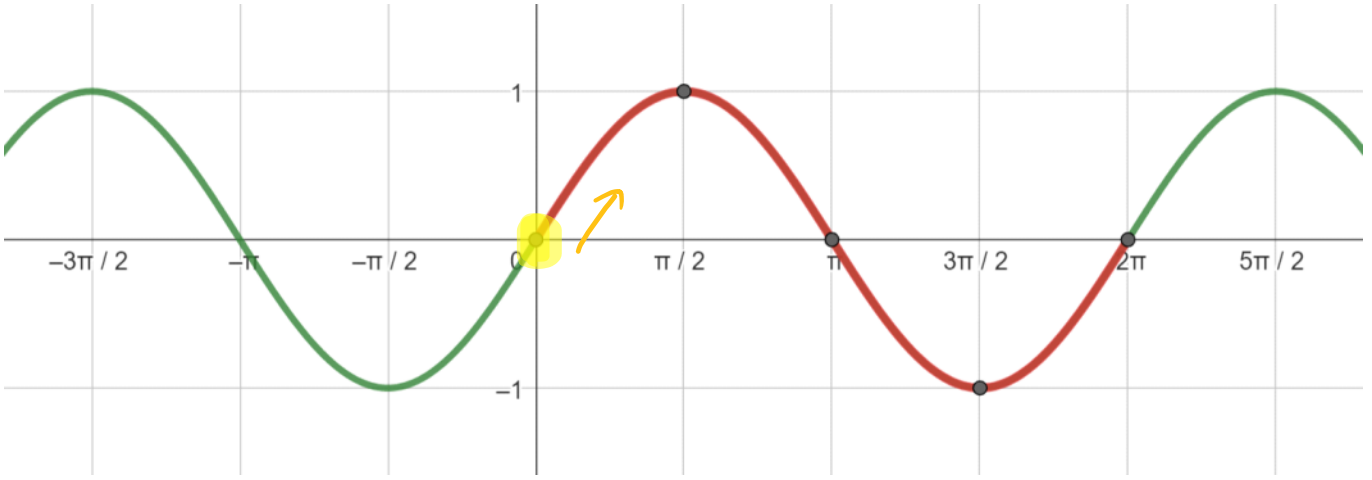
$y = \cos(x + \pi)$

At what x-values do these graphs have points of inflection?

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$x = \frac{\pi}{2} + \pi k, \; k \in \mathbb{Z}$

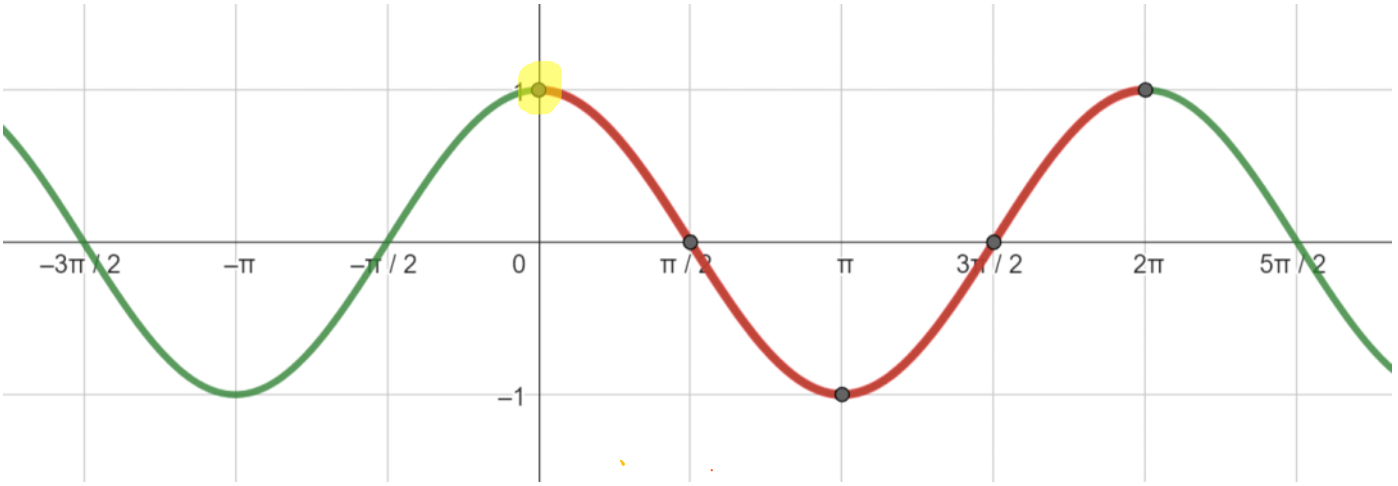
The Graph of $f(x) = \sin x$



Period: 2π Frequency: $\frac{1}{2\pi}$ Amplitude: 1

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$

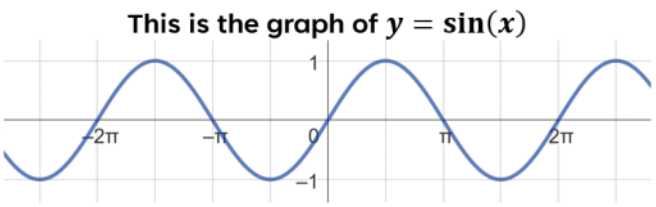
The Graph of $f(x) = \cos x$

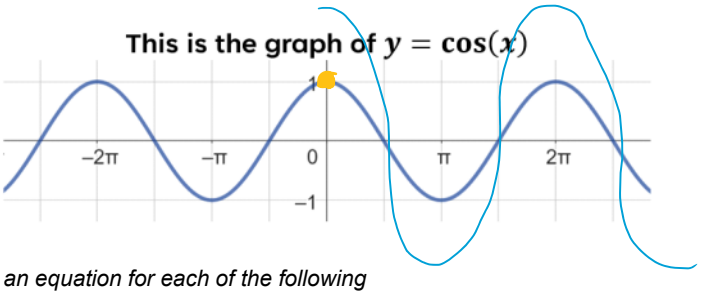


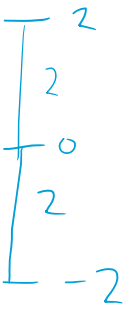
Period: 2π Frequency: Amplitude: 1

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$

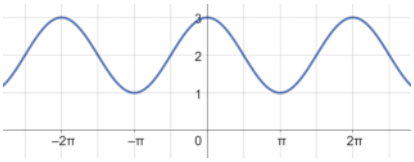
Complete the front and back of the notes



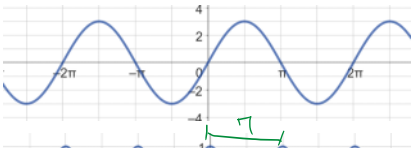




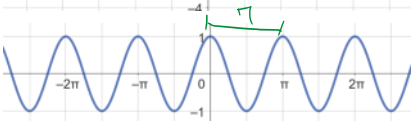
Apply your understanding of transformation of functions to try to find an equation for each of the following graphs



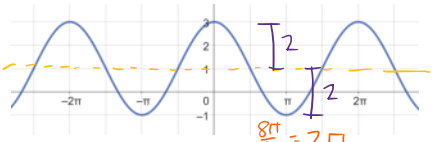
$y = \cos x + 2$



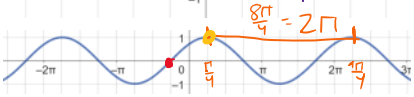
$y = 3\sin x$



$y = \cos(2x)$



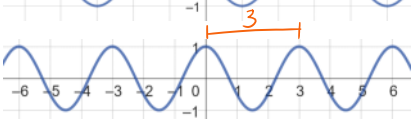
$y = 2\cos x + 1$



$y = \cos\left(x - \frac{\pi}{4}\right)$

$y = \sin\left(x + \frac{\pi}{4}\right)$

$y = -\sin x$



$y = \cos\left(\frac{2\pi}{3}x\right)$

$P = 2\pi \rightarrow P = 3$

horiz dilation by $\frac{3}{2\pi}$

$2\pi.$ =

Sinusoidal Functions: Functions that result from reflections, dilations, and translations performed on the parent sine or cosine function

$a\sin\big(b(x+c)\big)+d$ or $a\cos\big(b(x+c)\big)+d$

Dilations

Recall:

- $y = af(x)$ vertically dilates by a factor of a
- $y = f(bx)$ horizontally dilates by a factor of $\frac{1}{b}$

For $f(x) = a\sin(bx)$ and $g(x) = a\cos(bx)$, (with $a \neq 0$ and $b \neq 0$)

$\text{Amplitude} = |a|$ and $\text{Period} = \frac{2\pi}{|b|}$

So, to find a and b you can always choose

$a = \text{Amplitude}$ and $b = \frac{2\pi}{\text{Period}}$

Translations

$a\sin(b(x+c))+d$

Recall:

- $y = f(x + c)$ horizontally translates
- $y = f(x) + d$ vertically translates

These transformations will affect the phase and midline of the parent graphs.

Phase Shift = The Horizontal Translation (should make the argument of the sine or cosine zero)

Midline = The horizontal line halfway between the minimum and maximum of the function

To find c and d, you can always choose

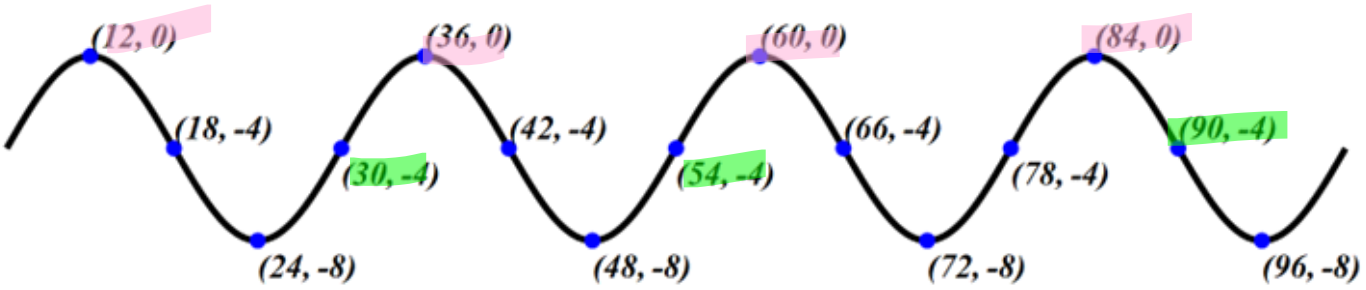
$c = -\text{start}$

$d = \text{midline}$

Let's Learn How to Find a, b, c, and d

https://qed-math.com/Applets/FRQ3BTrainer/index.html

Example 1



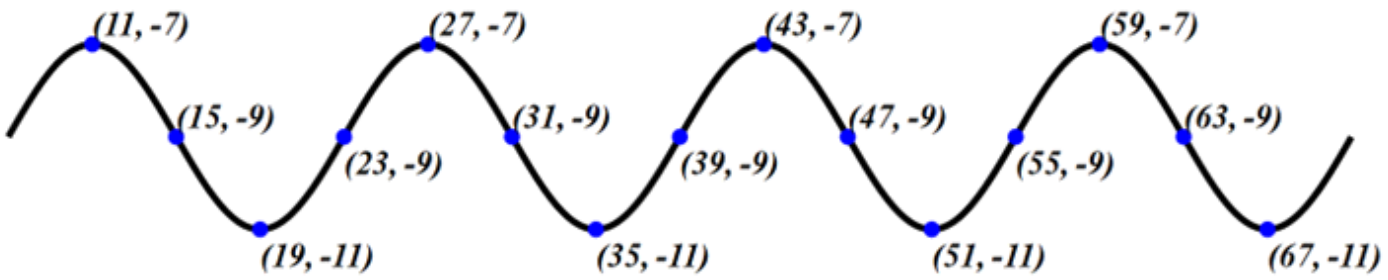
Write an equation of the form $f(x) = a\cos\big(b(x+c)\big) + d$

$f(x) = 4\cos\left(\frac{\pi}{12}(x-60)\right) - 4$

Write an equation of the form $f(x) = a\sin\big(b(x+c)\big) + d$

$f(x) = 4\sin\left(\frac{\pi}{12}(x-54)\right) - 4$

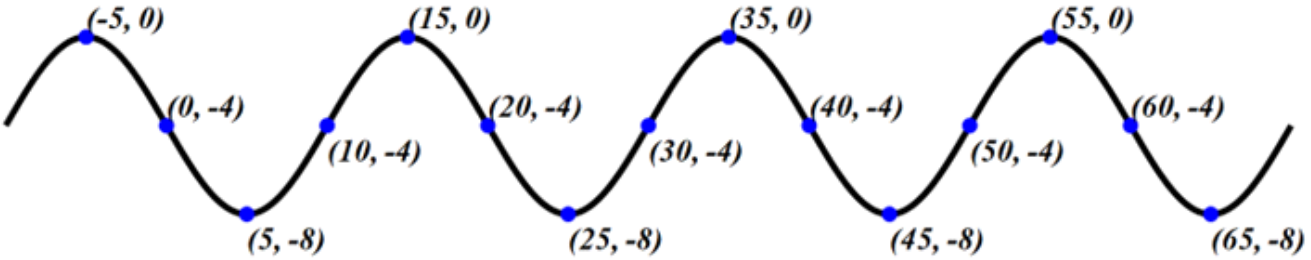
Example 2



Write an equation of the form $f(x) = a\sin\big(b(x+c)\big) + d$

Write an equation of the form $f(x) = a\cos\big(b(x+c)\big) + d$

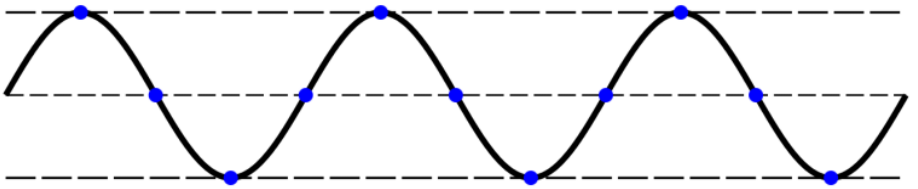
Example 3



Write an equation of the form $f(x) = a\sin\big(b(x+c)\big) + d$

Write an equation of the form $f(x) = a\cos\big(b(x+c)\big) + d$

Sketch $y = 2\sin\left(\frac{\pi}{8}(x+3)\right) + 1$



Midline:

Amplitude:

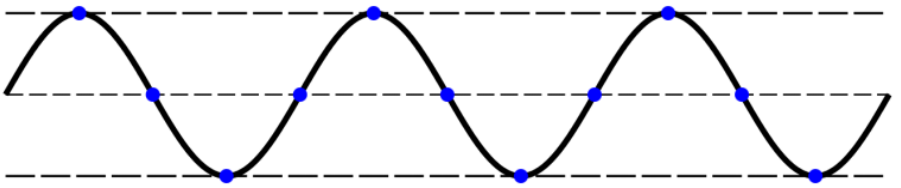
y-values

Period:

Quarter Period:

Phase Shift/Start:

Sketch $y = 3\cos(2x) - 7$



Midline:

Amplitude:

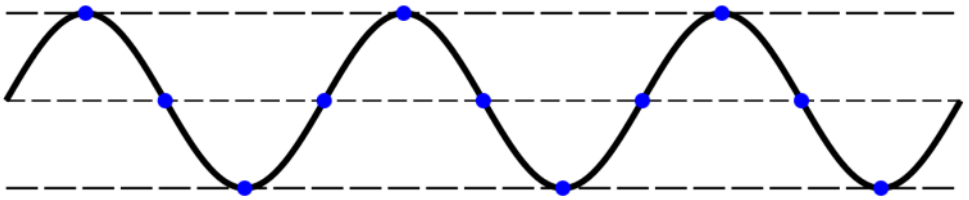
y-values

Period:

Quarter Period:

Phase Shift/Start:

$f(x) = 4 + 2\sin(3x + 3\pi)$



Midline:

Amplitude:

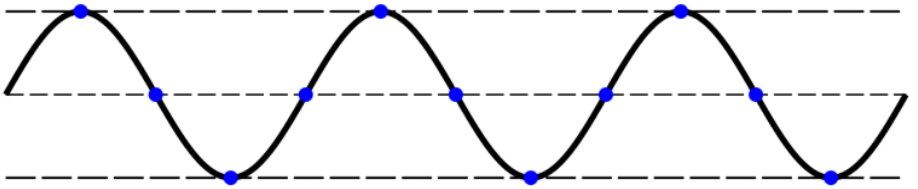
y-values

Period:

Quarter Period:

Phase Shift/Start:

$f(x) = 3\cos\left(\frac{\pi}{4}(x-1)\right) + 1$



Midline:

Amplitude:

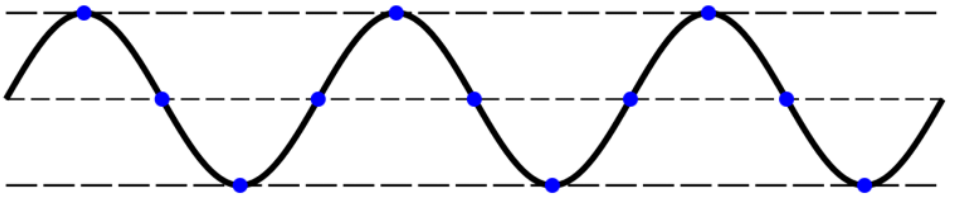
y-values

Period:

Quarter Period:

Phase Shift/Start:

$f(x) = 2\sin(4x - \pi) + 1$



Midline:

Amplitude:

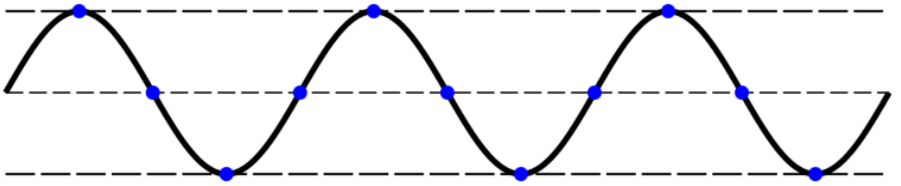
y-values

Period:

Quarter Period:

Phase Shift/Start:

$$f(x) = -2\cos\left(4\left(x - \frac{\pi}{4}\right)\right) + 1$$



Midline:

Amplitude:

y-values

Period:

Quarter Period:

Phase Shift/Start:

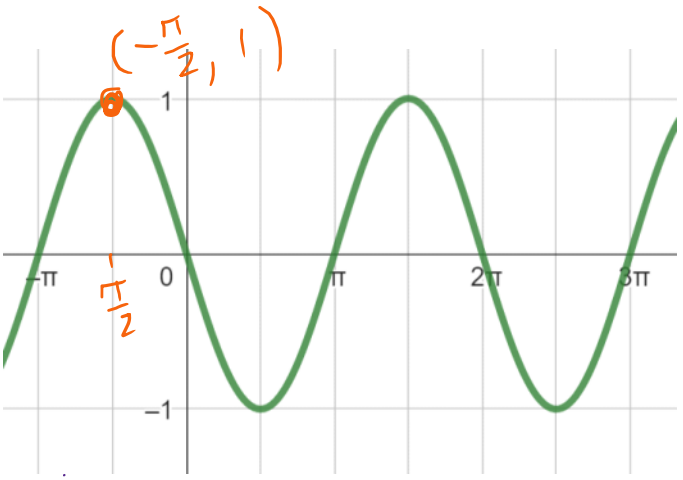
One Graph, Multiple Possible Equations

As the previous examples have shown, many sinusoidal functions have the same graph

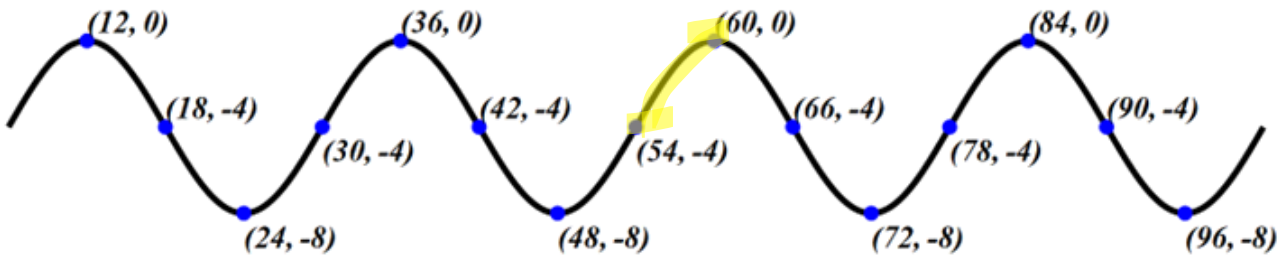
$y = -\sin(x)$

$y = \cos\left(x + \frac{\pi}{2}\right)$

$y = -\cos\left(x - \frac{\pi}{2}\right)$



Example 1



Write an equation of the form $f(x) = a\cos\big(b(x+c)\big) + d$ that uses a negative value of a.

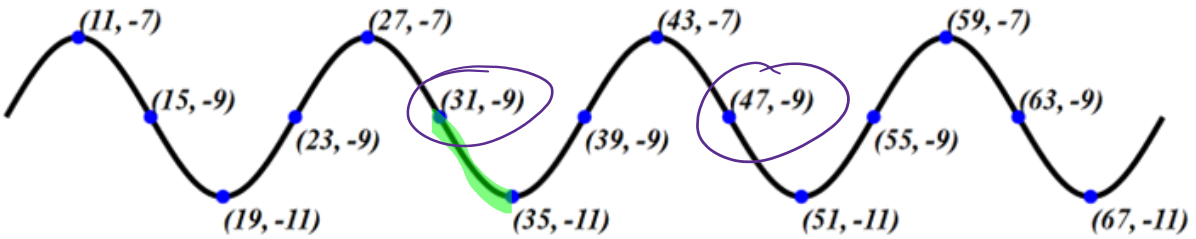
$-4\cos\left(\frac{\pi}{12}(x-48)\right)-4$

Over the interval $54 < x < 60$, $f(x)$ is...

(A) Positive and Increasing
(B) Positive and Decreasing
(C) Negative and Increasing
(D) Negative and Decreasing

Find an expression for all x-values at which this graph has points of inflection

Example 2



$-2\sin\left(\frac{\pi}{8}(x-47)\right)-9$

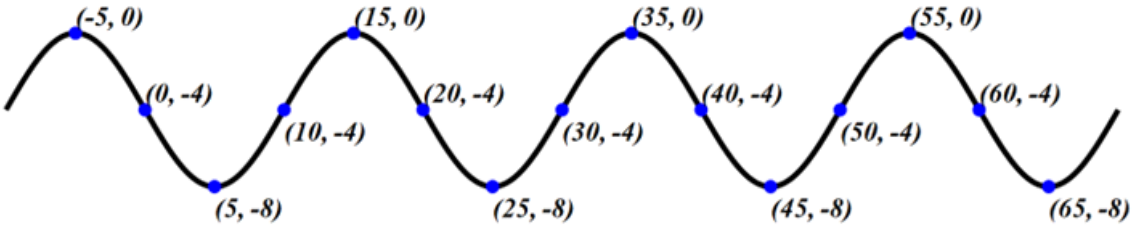
Write an equation of the form $f(x) = a\sin\big(b(x+c)\big) + d$ that uses a negative value of a.

Over the interval $31 < x < 35$, is the graph of $f(x)$ concave up or concave down? Over that interval, is the rate of change of $f(x)$ increasing or decreasing? f(x) is concave up

rate of change increasing

Find an expression for all x-values at which this graph has maxima.

Example 3



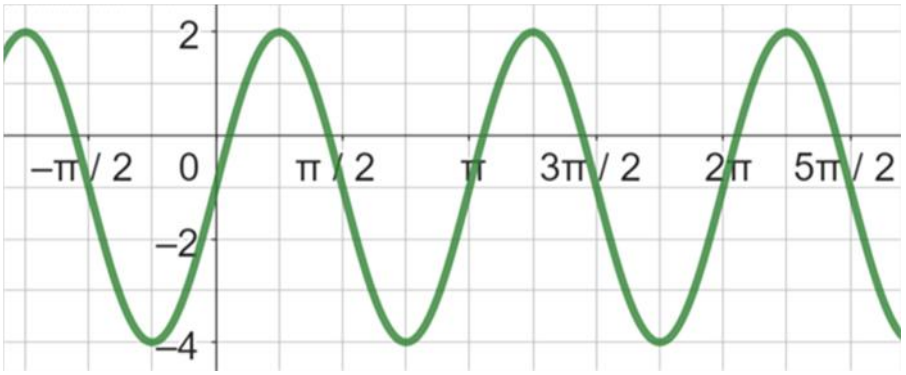
Write an equation of the form $f(x) = a\sin\big(b(x+c)\big) + d$ that uses a negative value of a.

Over the interval $45 < x < 50$, $f(x)$ is...
(A) Positive and Increasing
(B) Positive and Decreasing
(C) Negative and Increasing
(D) Negative and Decreasing

Over the interval $45 < x < 50$, is the graph of $f(x)$ concave up or concave down? Over that interval, is the rate of change of $f(x)$ increasing or decreasing?

Consider the graph shown to the right. Which of the following is NOT a possible equation of this graph

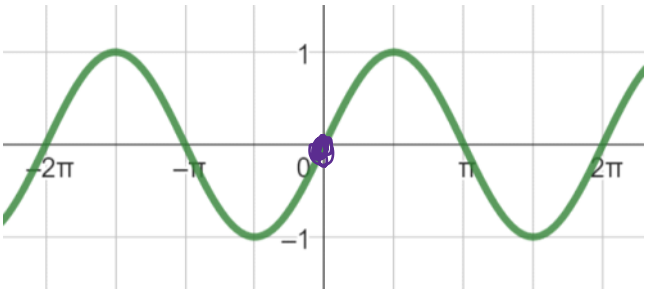
(A) $y = -1 + 3\sin(2x)$
(B) $y = -1 - 3\sin(2x - \pi)$
(C) $y = -1 + 3\sin\big(2(x - \pi)\big)$
(D) $y = -1 + 3\cos\left(2x - \frac{\pi}{2}\right)$
(E) $y = -1 + 3\cos\left(2\left(x + \frac{\pi}{4}\right)\right)$
(F) $y = -1 - 3\cos\left(2\left(x - \frac{3\pi}{4}\right)\right)$



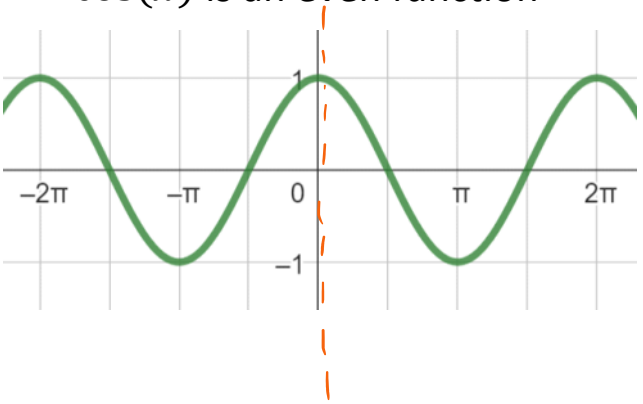
Even/Odd

origin

$\sin(x)$ is an odd function



$\cos(x)$ is an even function



One of the following functions is even. One of the following functions is odd. One of the following functions is neither. Determine which is which.

$f(x) = 3\sin\left(x - \frac{\pi}{2}\right)$ $g(x) = 4 - \sin(2x)$ $h(x) = \cos\left(x + \frac{\pi}{2}\right)$

$b = 1$

$f(\pi) = 3\sin\left(\pi - \frac{\pi}{2}\right) = 3\sin\left(\frac{\pi}{2}\right) = 3$

$f(-\pi) = 3\sin\left(-\pi - \frac{\pi}{2}\right) = 3\sin\left(\frac{-3\pi}{2}\right) = 3$

amplitude = 3

midline = 0

$\frac{2\pi}{\text{Period}} = 1$

Period $= 2\pi$